7135 Janes Avenue
Woodridge, Illinois 60517
www.enengineering.com

NEW EARTH POTENTIAL EQUATIONS AND APPLICATIONS

Rogelio de las Casas
EN Engineering
7135 Janes Avenue
Woodridge, Illinois 60517

ABSTRACT

The distance between the groundbed and protected structure should be considered during the design of an impressed current cathodic protection system. This distance is determined using the ratio between the potential change in the earth due to the groundbed (buried electrodes) and the groundbed potential to remote earth. Currently the equations used to calculate the potential change in the earth due to buried electrodes are for single vertical or horizontal electrodes only. This paper presents new equations for the determination of potential changes in the earth due to the groundbed. Unlike the currently used equations, the developed equations work for groundbeds with any number of anodes, either vertical or horizontal, and work for groundbeds with varied anode spacing. These equations provide a method to calculate the groundbed's area of influence, aiding in the determination of optimum groundbed location. The equations also apply to any metallic structure in an electrolyte that can be affected by direct current flow.

Keywords: anodes to remote earth potentials, cathodic protection design, stray current, direct current interference, anodes area of influence.

INTRODUCTION

The minimum distance between the groundbed and protected structure should be considered during the design of a cathodic protection system. Currently, the equations used to calculate the potential change in the earth due to buried electrodes are for single vertical or single horizontal electrodes only.

Date	Version	Approved	Comments

This paper presents new equations for determining the potential in the earth due to the groundbed. Unlike the currently used equations, these newly developed equations work for groundbeds with any number of anodes, either vertical or horizontal, and work for groundbeds with varied anode spacing and any distance between the surface of the earth and the top of the anodes. These equations provide a method to calculate the groundbeds area of influence, aiding in the determination of optimum groundbed location. These new equations also apply to any metallic structure in an electrolyte that can be affected by direct current flow.

BACKGROUND

The earth potential to ground equations presented by Erling D. Sunde in his book “Earth Conduction Effects in Transmission Systems” are for single anode installations either vertical or horizontal. The coordinate axes for equation deductions are in the center of the electrode, and the electrode is placed in a plane formed by the axis x and y.

The theory supporting Sunde’s equation of one electrode is as follows:[1]

1) The potential due to a buried electrode in the ground surface is calculated taking into account the current of the electrode and the current due to its image in the surface of the ground.

2) In the initial formula, the potential of just one charged point in position (u, 0) over another remote point in (x, y), where z=0, is determined by Equation 1 below. Also noteworthy is the number “2” in the formula. This is included since the electrical charge of the point’s image is being considered as well.

$$V := \frac{2Ie \cdot \rho}{4 \cdot \pi \cdot r} \qquad (1)$$

Where:

V = potential of the point in space at distance r from the charge (volts)

Ie = current of the charged point (amperes)

ρ = soil resistivity (ohm*m)

r = distance between the charged point and the point where the potential will be determined. “r” is determined by the following equation:

$$r := \sqrt{(x-u)^2 + y^2} \qquad (2)$$

The calculation to determine the potential for an electrode with the origin of coordinates in the center, and with length equal to L meters is:

$$V(x,y) := \frac{Ie}{2\pi L}\cdot\rho\cdot\int_{\frac{-L}{2}}^{\frac{L}{2}} \frac{1}{\sqrt{(x-u)^2 + y^2}}\,du \qquad (3)$$

The charge of the electrode is considered to be concentrated at the electrode axis. This statement is congruent with the Newtonian potential theory, where the length of the electrode is much larger than its diameter, and is also more accurate when the point where the potential is determined is very far from the electrode axis[2].

Remote earth

Remote earth is a location on the earth far enough from the affected structure that the soil potential gradients associated with currents entering the earth from the affected structure are insignificant.

When an existing groundbed is receiving current from a rectifier, the potential profile can be measured by placing a reference cell connected to the positive terminal of a multimeter over the center of the groundbed, and using another reference cell connected to the negative terminal of the multimeter located at remote earth. This configuration is shown in Figure 1. By measuring using this configuration, the magnitude of the readings will be positive. Moving the first electrode away from the groundbed, the potential profile of the groundbed will be obtained with respect to the distance of the cell from the groundbed.

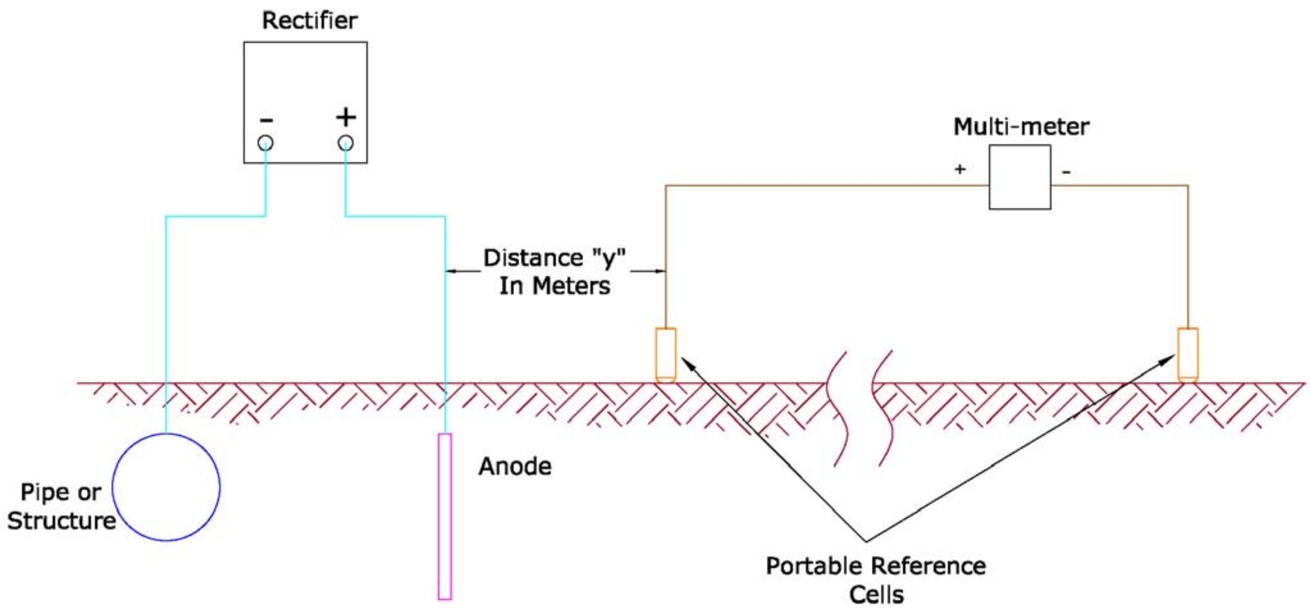

Figure 1: Potential Profile Measured Using Two Reference Cells

To measure the remote earth distance needed for potential in the ground measurements, the positive terminal of the multimeter is connected to the groundbed header cable and the negative terminal is connected to a reference cell, as shown in Figure 2. The potential is measured by moving the reference cell away from the groundbed. The location where there is no longer a significant change in the potential reads is the location of remote earth.

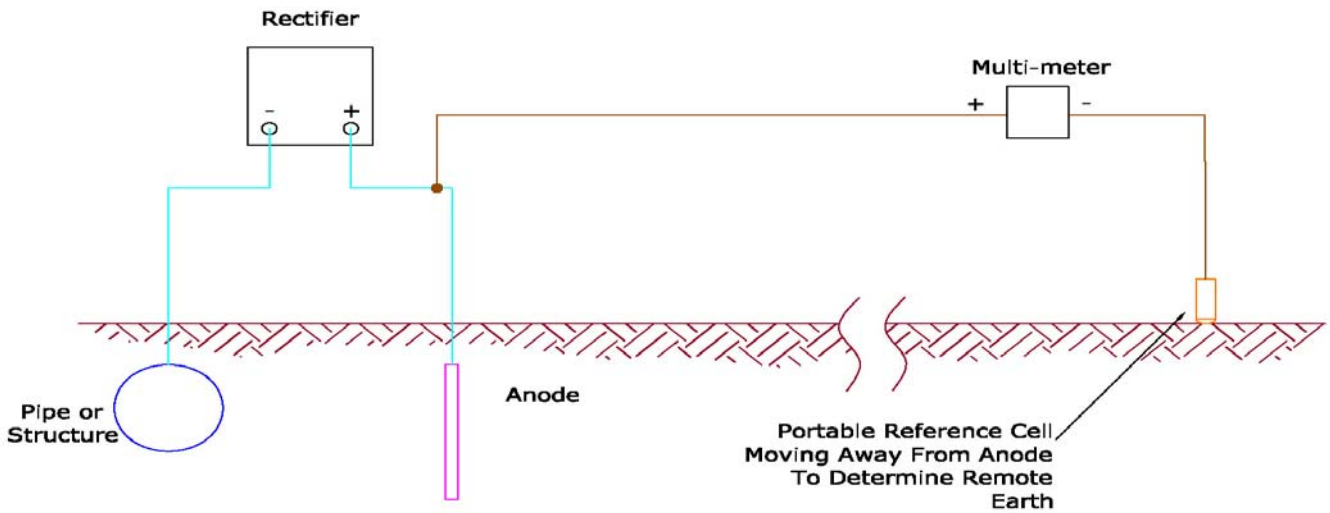

Figure 2: Remote Earth Distance Measured Using One Reference Cell

In order to calculate the potential of a point (x, y, z) in the space of infinite extension (where infinite extension means that the distance between the electrode and the point where the potential is measured is much larger than the electrodes dimensions) with respect to the electrode length, it is assumed that the electrode current density is distributed evenly over the entire surface of the electrode. This is called the average leakage current. Using this information, the potential caused by the entire electrode surface can be calculated. These steps can also be followed in Von Baeckmann and Schwenk's book "Handbook of Corrosion"[3].

In the above theory, the supposed origin of coordinates is at the center of the anode, and the integration limits are between + and – ½ the length of the anode with just one anode affecting the point (x, y, z).

The remainder of this paper displays the author's newly developed practical equations, following the same Sunde's integration method, but for cases with multiple horizontal or multiple vertical electrodes in groundbed configurations, with the origin of coordinates at the center of the structure to be protected (i.e. pipeline center line), and at various anode spacing.

NEW CALCULATION PROCEDURE FOR GROUND POTENTIALS

Potential changes in the ground are a direct result of the installation and activation of cathodic protection anodes as part of a cathodic protection system. Impressed current system anodes and sacrificial anodes provoke the same potential profile in the soil. The localization of remote earth is important in the determination of the location for groundbed installations.

Occasionally, the cathodic protection of a buried structure is best designed using anodes installed close to the structure. With the ability to calculate the maximum distance in the soil that is under the influence of the anodes, the determination of adequate protection for the structure is obtainable. In these instances, the current is more concentrated over the protected structure, with the magnitude of current arriving at the pipe surface where the pipe is inside the ground bed's area of influence. See Figure 3 for an example of this situation.

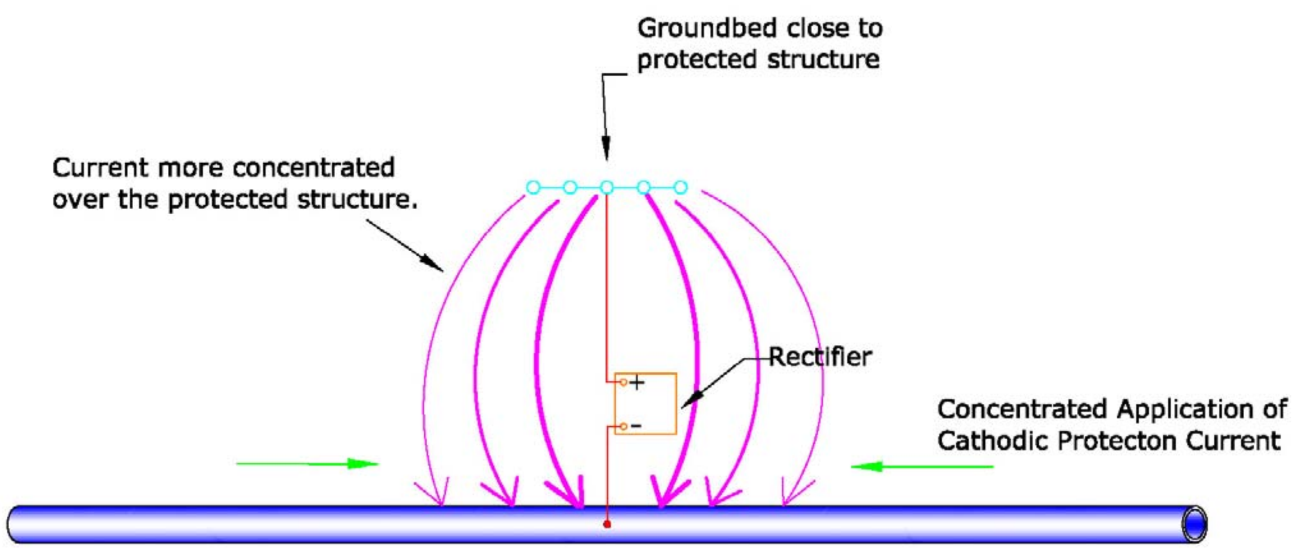

Figure 3: Groundbed close to protected structure.

The following figure, Figure 4, is an example of Close Interval Survey data when the groundbed is located close to the structure. Note that the potential readings indicate a concentration of current on the pipe. This type of installation requires additional groundbed installations in order to adequately protect the pipe. A more desirable situation would have a less concentrated or greater area (i.e. spread) of current distribution, resulting in a larger area of protection on the structure from one protection source (i.e. groundbed) with the same amount of current.

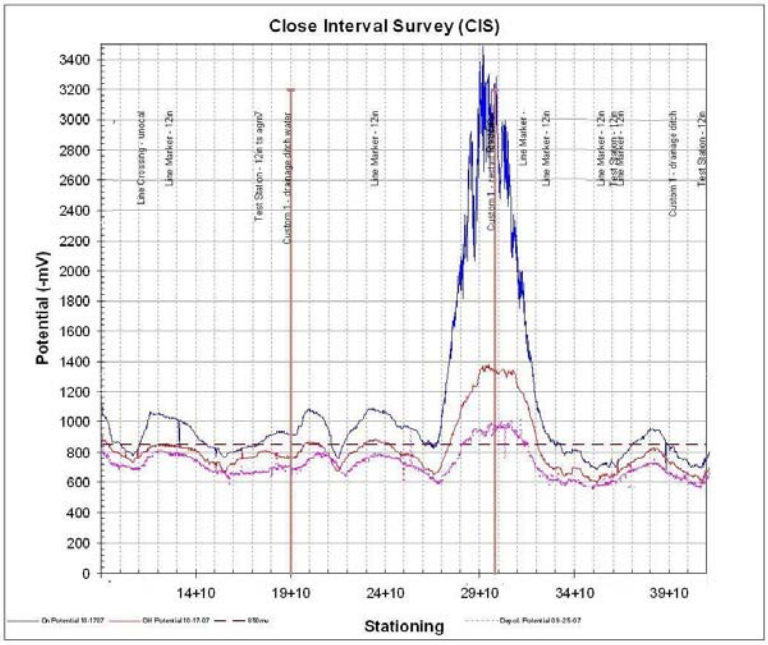

Figure 4: Example of Close Interval Survey data with the groundbed location close to the protected structure.

In most cases, cathodic protection systems are designed to reach as much structure surface as possible with the groundbed, while at the same time taking into consideration foreign structures and the possibility of interference on those structures. With a design of this type, the current flowing through each individual electrolytic path has a similar magnitude, as seen in Figure 5.

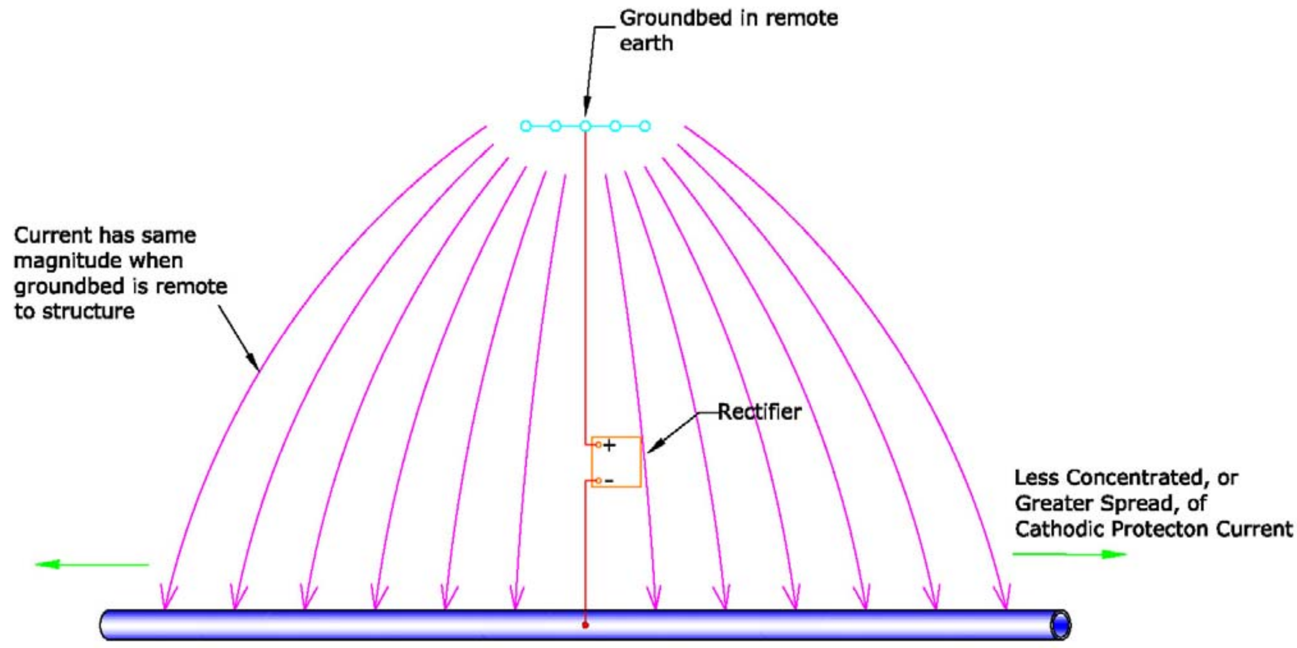

Figure 5: Groundbed in remote location.

The new design equations updated Sunde's theory in order to consider any number of anodes. These new equations take into account anodes which are at a minimum distance from the center of the structure while moving the origin of coordinates from the center of the one anode to the center of the structure to be protected. This minimum distance, the remote distance from the protected structure, can be calculated afterward. This minimum distance for the location of the groundbed allows for the magnitude of current to be even across the surface of the protected structure.

To assist in design calculations, a program was also developed in Mathcad® for calculating the remote distance between the groundbed and structure to be protected. Following the ideas presented in the NACE CP level IV course book[4], the potential to remote earth created by the anode in the ground is compared with the current multiplied by the groundbed resistance to remote earth. The distance regarded as 'remote' for a practical point of view is the distance where the potential to remote earth developed by the groundbed in the ground is equal to or less than 5% of the groundbed potential to remote earth. Again, the equations for several anodes are used instead of the potential of just one anode. Based on the output from the equations, graphs of the potential developed in the ground due to the pipe and groundbeds are developed with regard to remote earth. These graphs also aid in presenting how the groundbed influences the earth around it.

MATHEMATICAL AND PHYSICAL CONSIDERATIONS FOR INTEGRATION

The mathematical and physical conditions under which these new equations were developed are:

- The first anode will be ξ meters from the pipeline center line; where ξ is much greater than the pipeline diameter or structures dimensions and is the distance defined as remote earth distance.
- The earth potential is calculated considering the electrode's current and the current due to the electrode's image in the earth surface.
- The anodes run in the y-z plane.
- As a base formula for integration, the potential of just one charged point (x_0, y_0, z_0) over another point (x, y, z) is determined, where the point (x, y z) is localized in the earth at a remote distance of (x_0, y_0, z_0). In the particular case when the potential is determined in the surface of the earth z is zero meters.
- The definition of remote earth is a location on the earth far enough from the affected structure that the soil potential gradients associated with currents entering the earth from the affected structure are insignificant.
- The soil resistivity around the electrodes is considered uniform.
- The action of several electrodes is cumulative.
- The x axis is the pipeline or structure axis.
- The y axis runs perpendicular to the pipeline with a value of 0 meters at the pipeline centerline and is parallel to the groundbed.
- The z axis runs perpendicular to the earth surface, with a value of 0 meter at the surface of the earth, over the pipeline centerline.
- u runs in the electrode axis.

The constants in the equations are the following:

- Distance between consecutive anodes - **S** in meters.
- Total number of anodes - **N**.
- Anode's length including coke backfill - **L** in meters.
- Depth of the top of the anode - **t** in meters.
- Current applied at the groundbed - **Idis** in Amperes.
- Average soil resistivity at the groundbed location - **ρ** in ohm*m
- Distance between the first anode and the origin of coordinates **ξ** in meters.

PRACTICAL APPLICATIONS OF EQUATIONS

The following three typical situations give practical examples of how these newly developed equations can be applied.

Situation 1

Horizontal anodes in a line, perpendicular to the structure, but in individual locations. Figure 6 shows anodes in this type of configuration. These anodes are not in the same coke horizontal column. This applies for congested areas where several horizontal anodes are needed, but the available space is separated by driveways or other structures preventing the installation of the anodes in one horizontal column.

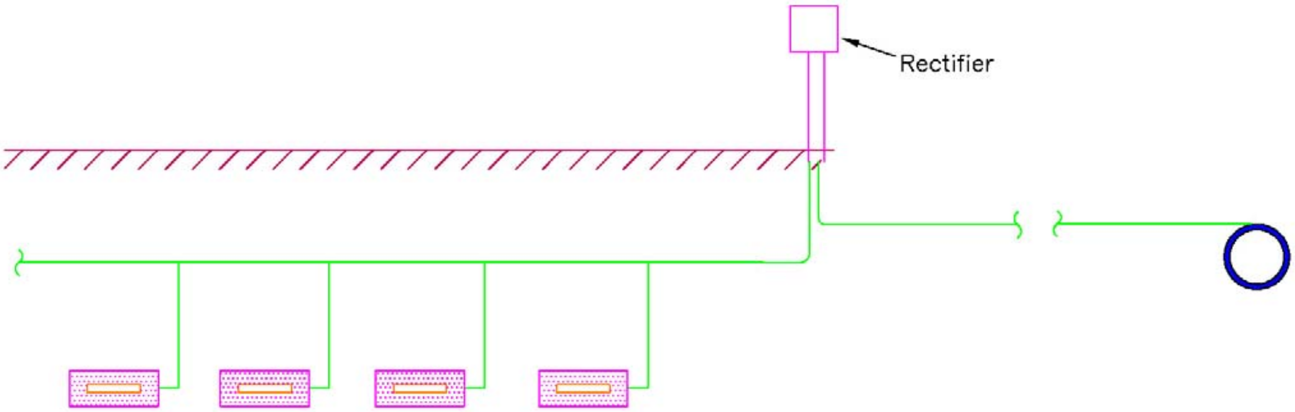

Figure 6: Individual horizontal anodes in a line perpendicular to a pipeline.

The integral to be calculated for one anode is:

$$\mathrm{Uremot}(x,y,z) := \frac{\mathrm{Idis}}{4\pi}\cdot\frac{\rho}{L}\cdot\int_0^L \left[\frac{1}{\sqrt{(x)^2+(y-u-\xi)^2+(z+t)^2}} + \frac{1}{\left[\sqrt{(x)^2+(y-u-\xi)^2+(z-t)^2}\right]}\right] du \quad (4)$$

The general equation to determine the potential in any point of the earth, including points deep in the earth is:

$$\mathrm{Uremo}(x,y,z) := \frac{\mathrm{Idis}}{4\pi}\cdot\frac{\rho}{L}\cdot\left[\ln\left[\frac{y-\xi+\sqrt{x^2+(y-\xi)^2+(t+z)^2}}{y-L-\xi+\sqrt{x^2+(y-L-\xi)^2+(z+t)^2}}\right] + \ln\left[\frac{y-\xi+\sqrt{x^2+(y-\xi)^2+(z-t)^2}}{y-L-\xi+\sqrt{x^2+(y-L-\xi)^2+(z-t)^2}}\right]\right] \quad (5)$$

The equation considering all the anodes in a groundbed configuration is:

$$\mathrm{Uremo}(x,y,z) := \sum_{p=0}^{N-1}\left[\frac{\mathrm{Idis}}{4\pi\cdot N}\cdot\frac{\rho}{L}\cdot\left[\ln\left[\frac{y-\xi+\sqrt{x^2+(y-\xi-p\cdot S)^2+(t+z)^2}}{y-L-\xi+\sqrt{x^2+(y-L-\xi-p\cdot S)^2+(z+t)^2}}\right] + \ln\left[\frac{y-\xi+\sqrt{x^2+(y-\xi-p\cdot S)^2+(z-t)^2}}{y-L-\xi+\sqrt{x^2+(y-L-\xi-p\cdot S)^2+(z-t)^2}}\right]\right]\right] \quad (6)$$

This equation can be used to determine the influence over buried structures such as gas or oil wells, buried tanks, buried pipelines, and any buried metallic structure.

Now consider that just the potential in the surface of the earth is needed. The potential can be measured with two reference cells and a high resistance multimeter.

The equation considering all the anodes is:

$$Uremot(x,y) := \sum_{p=0}^{N-1}\left[o\cdot\ln\left[\frac{\sqrt{x^2+(y-p\cdot S-\xi)^2+t^2}+y-p\cdot S-\xi}{\sqrt{(x)^2+(y-p\cdot S-L-\xi)^2+t^2}+y-p\cdot S-L-\xi}\right]\right] \tag{7}$$

where: $$o := \frac{Idis\cdot\rho}{2\cdot\pi\cdot N\cdot L}$$

The equation for Situation 1 helps to solve for the location of remote earth ξ, which is the placement location for the first anode in the horizontal groundbed line. The groundbed is comprised of horizontal anodes placed perpendicular to the structure in their own coke breeze backfill (i.e. not a continuous column of backfill).

Example variables used to determine the distance of remote earth for the case study above are:

S = 4.572 meters	N =10
L =2.134 meters	T =1.524 meters
Idis =12 Amperes	ρ = 87 ohm*m

By performing the calculations, the minimum distance perpendicular from the pipe where the groundbed can be installed remote is 112 meters.

Situation 2

Horizontal anodes in individual locations, in parallel distribution with respect to the structure. Figure 7 shows a plan view for this type of installation. The anodes are parallel to each other and to the pipeline, but at a remote distance.

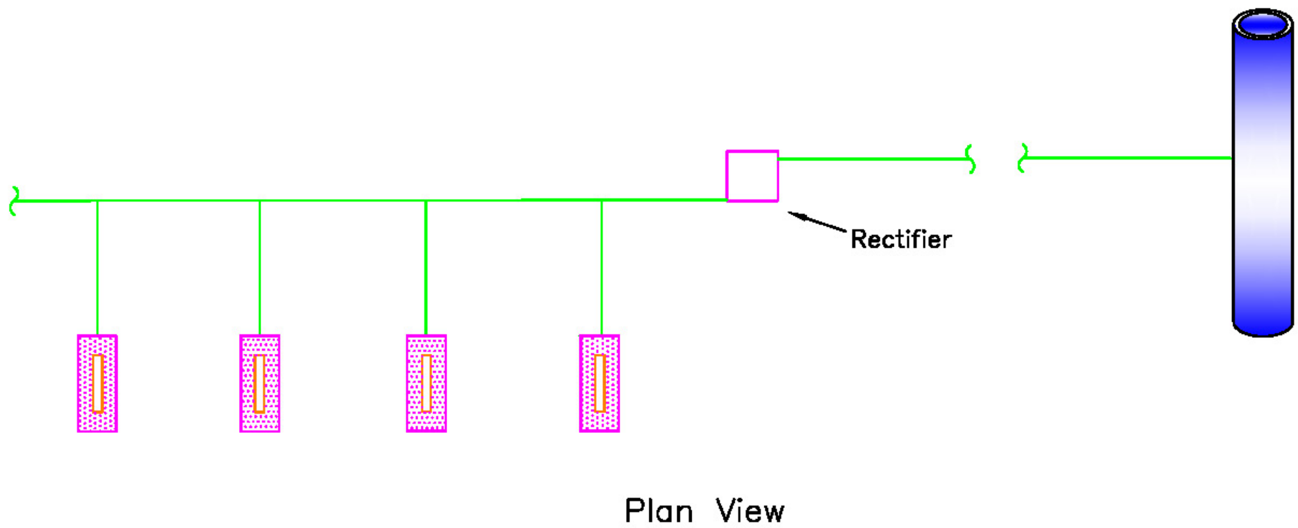

Figure 7: Plan view of individual horizontal anodes installed perpendicular to the pipe.

The integral is:

$$\mathrm{Uremot}(x,y,z) := \frac{\mathrm{Idis}}{4\pi}\cdot\frac{\rho}{L}\cdot\int_0^L \left[\frac{1}{\sqrt{(x-u)^2+(y-\xi)^2+(z+t)^2}} + \frac{1}{\left[\sqrt{(x-u)^2+(y-\xi)^2+(z-t)^2}\right]}\right] du \tag{8}$$

The potential equation for single anode acting over any point in the earth is:

$$\mathrm{Uremot}(x,y,z) := \frac{\mathrm{Idis}}{4\pi}\cdot\frac{\rho}{L}\cdot\left[\ln\left[\frac{x+\sqrt{x^2+(y-\xi)^2+(t+z)^2}}{x-L+\sqrt{(x-L)^2+(y-\xi)^2+(z+t)^2}}\right] + \ln\left[\frac{x+\sqrt{x^2+(y-\xi)^2+(z-t)^2}}{x-L+\sqrt{(x-L)^2+(y-\xi)^2+(z-t)^2}}\right]\right] \tag{9}$$

The potential equation for multiple anodes, in a groundbed configuration, for potential in any point in the earth is:

$$\mathrm{Uremot}(x,y,z) := \sum_{p=0}^{N-1}\left[\frac{\mathrm{Idis}}{4\cdot\pi\cdot N}\cdot\frac{\rho}{L}\cdot\left[\ln\left[\frac{x+\sqrt{x^2+(y-\xi-p\cdot S)^2+(t+z)^2}}{x-L+\sqrt{(x-L)^2+(y-\xi-p\cdot S)^2+(t+z)^2}}\right] + \ln\left[\frac{x+\sqrt{x^2+(y-\xi-p\cdot S)^2+(z-t)^2}}{x-L+\sqrt{(x-L)^2+(y-\xi-p\cdot S)^2+(z-t)^2}}\right]\right]\right] \tag{10}$$

The equation to determine the potential in the surface of the earth, where z=0 m:

$$\mathrm{Uremot}(x,y) := \sum_{p=0}^{N-1}\left[o\cdot\ln\left[\frac{\sqrt{x^2+(y-p\cdot S-\xi)^2+t^2}+x}{\sqrt{(x-L)^2+(y-p\cdot S-\xi)^2+t^2}+x-L}\right]\right] \tag{11}$$

where:

$$o := \frac{\mathrm{Idis}\cdot\rho}{2\cdot\pi\cdot N\cdot L}$$

The equation for Situation 2 helps to determine the location of remote earth ξ, which is the location for the placement of first horizontal anode. The anodes in the horizontal ground bed line are placed horizontally and parallel to the structure, each in their own coke breeze backfill.

Example variables used to determine the distance of remote earth for the case study above are:

S = 4.572 meters	N =10
L =2.134 meters	T =1.524 meters
Idis =12 Amperes	ρ = 87 ohm*m

The minimum distance perpendicular from the pipe where the groundbed can be installed and be at remote earth is 110 meters.

A graphical approach of the above equation including the potential to remote earth of the protected structure is shown in Figure 8 for the case of a pipeline with good coating and the anodes installed horizontally. The potential graph is for potentials in the surface of the earth.

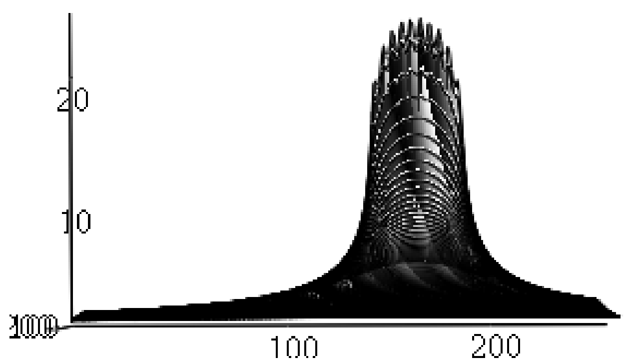

Figure 8: Graph of combined potentials for a well coated pipeline and groundbed

Situation 3

Vertical anodes in a groundbed perpendicular to the structure.

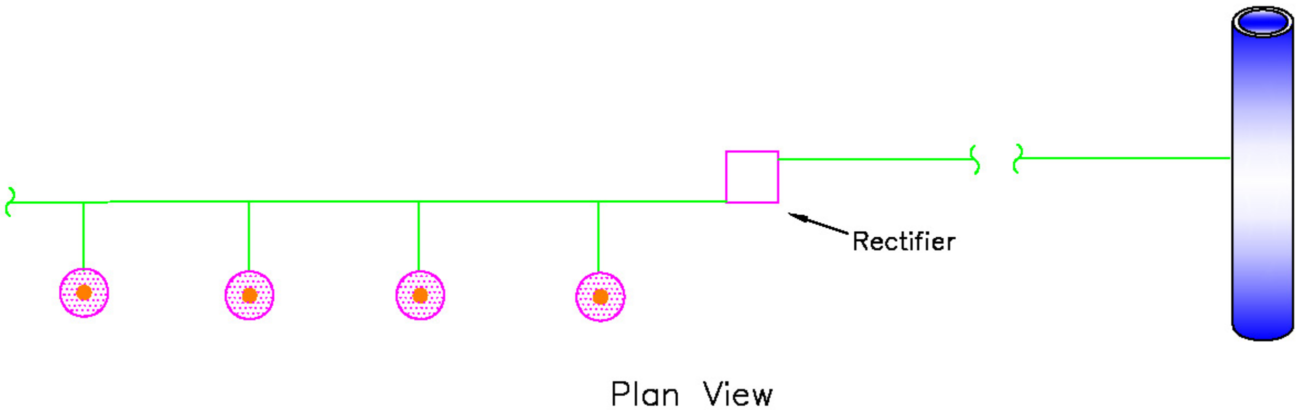

Figure 9: Plan view of vertical anodes installed perpendicular to the pipe.

The integral is:

$$\mathrm{Uremot}(x,y,z) := \frac{\rho}{4\pi}\cdot\frac{I(0)}{L}\cdot\int_0^L \left[\frac{1}{\sqrt{(x)^2+(y-\xi)^2+(z+u+t)^2}}+\frac{1}{\sqrt{(x)^2+(y-\xi)^2+(z-u-t)^2}}\right] du \tag{12}$$

The potential equation for the case of single vertical anode, acting over the earth is:

$$\mathrm{Uremot}(x,y,z) := \frac{\rho}{4\pi}\cdot\frac{I(0)}{L}\cdot\left[\ln\left[\frac{L+t+z+\sqrt{x^2+(y-\xi)^2+(z+L+t)^2}}{z+t+\sqrt{x^2+(y-\xi)^2+(z+t)^2}}\right]+\ln\left[\frac{z-t+\sqrt{x^2+(y-\xi)^2+(z-t)^2}}{z-t-L+\sqrt{x^2+(y-\xi)^2+(z-t-L)^2}}\right]\right] \tag{13}$$

The equation for multiple vertical anodes in groundbed configuration is:

$$\mathrm{Uremot}(x,y,z) := \sum_{p=0}^{N-1}\left[\frac{\rho}{4\pi\cdot N}\cdot\frac{I(0)}{L}\cdot\left[\ln\left[\frac{L+t+z+\sqrt{x^2+(y-p\cdot S-\xi)^2+(z+L+t)^2}}{z+t+\sqrt{x^2+(y-p\cdot S-\xi)^2+(z+t)^2}}\right]+\ln\left[\frac{z-t+\sqrt{x^2+(y-p\cdot S-\xi)^2+(z-t)^2}}{z-t-L+\sqrt{x^2+(y-p\cdot S-\xi)^2+(z-t-L)^2}}\right]\right]\right] \tag{14}$$

The equation to determine the potential in the surface of the earth where z=0 m is:

$$Uremot(x,y) := \sum_{p=0}^{N-1}\left[o\cdot \ln\left[\frac{L + t + \sqrt{x^2 + (y - p\cdot S - \xi)^2 + (L + t)^2}}{t + \sqrt{(y - p\cdot S - \xi)^2 + t^2}}\right]\right] \quad (15)$$

where: $o := \frac{Idis\cdot \rho}{2\cdot \pi \cdot N\cdot L}$

The equation for Situation 3 helps to calculate the location of remote earth ξ for placement of the first anode. The anodes in the horizontal ground bed line are placed vertically and perpendicular to the structure.

Example of an observation gas well and vertical shallow groundbed:

S = 6.096 meters

N =3

L = 3.658 meters

T =2.438 meters

Idis = 10 Amperes

ρ = 20 ohm*m

Lw = 100 m (well length)

Minimum distance to install the groundbed is 56 m.

The following graph in Figure 10 shows the combined potential in the ground due to the ground bed and the gas well together. The distance calculated (56 m) is from the first anode of the groundbed to the center of the well.

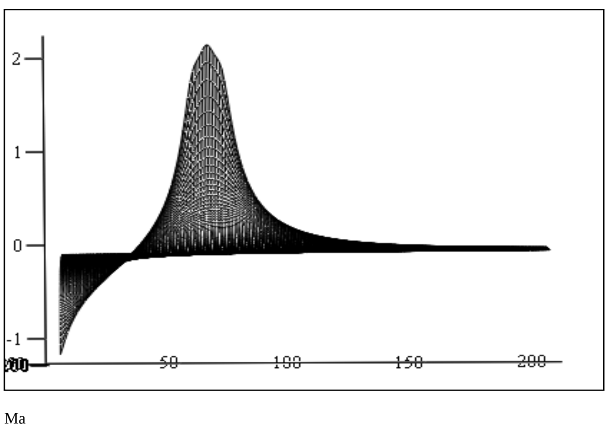

Figure 10: Graph of combined potentials for observation gas well and groundbed

RESULTS

These equations have been used by the author for design applications for the past three years. In each case, the location for the ground bed was chosen based on the remote earth calculations. One application involved installations to protect gas injection wells. Based on measurements taken during the commissioning of these groundbeds, all wells are receiving protection even though they are located far from the groundbed and have other wells located between the protected wells and the groundbed. Other locations have resulted in the location of remote easements for the groundbed installation, with protection levels covering the extent of the structure.

In the case of new pipelines with purchased easements, these newly developed equations aided in the determination of the groundbed location prior to the pipeline installation, allowing for the pipeline owner to not only budget for the additional easement, but to procure the easement during the initial stage of the project. Using the graphical output of these equations, the reluctance of the pipeline owner to purchase additional easements was overcome when they had a better understanding of the affect of the cathodic protection design not only on their pipeline, but also on foreign structures in the area.

CONCLUSION

The efficiency of cathodic protection systems depends on the accuracy of groundbed location with respect to protected structure and foreign structures. If the intent of the cathodic protection system is to protect a large extension of the structure, like a pipeline or well casing for example, and the groundbed is not truly ‘remote’ to the structure, the possibility exists for sections of the protected structure to receive less than adequate protection and for the parts of the structure closest to the groundbed to receive excessive protection. In the same manner, foreign structures inside the influence of the groundbed will be susceptible to stray current.

The developed equations allow the calculation of the area of influence of groundbeds in the ground in order to facilitate the determination of the right location for the groundbed installation. Graphical presentations illustrate the area of influence allowing for the selection of the appropriate location for the groundbed installation and to facilitate the understanding of influence of the cathodic protection system.

These new equations have important distinctions from the existing ones:

- They were determined regarding anodes in groundbed configuration.
- They can calculate the potential not just in the surface of the earth, but also in any point deep in the earth.

REFERENCES

1. Oliver Dimon Kellogg, “Foundations of Potential Theory”, 1953, chapter 1
2. Sunde, Erling D., “Earth Conduction Effects in Transmission Systems”, 1968, Chapters 2 and 3
3. Von Baeckmann and Schwenk; “Handbook of Cathodic Protection”, 3rd edition, Chapter 24
4. NACE International, “NACE Cathodic Protection Course Level 4”, 2004; Chapter 6